Spacetime quantization induced by axial currents

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Abstract

In the present contribution we show that the introduction of a conserved axial current in electrodynamics can explain the quantization of electric charge, inducing at the same time a dynamical quantization of spacetime.

I. INTRODUCTION

In 1931 Dirac proposes an electromagnetic theory with magnetic monopoles [1], whose appeal is mainly connected to the possibility of explaining the quantization of electric charge. In spite of this undeniable theoretical appeal, in Dirac's theory one is faced with a symmetry problem: the terms responsible for the magnetic monopole in the generalized Maxwell equations violate their symmetry under space and time reversal.

In this work we investigate the introduction of a new conserved current, namely an axial current, which presents the following differences as compared to the previously proposed vector magnetic current: (i) the resulting theory preserves space and time inversion invariance; (ii) besides the charge quantization, we can obtain a dynamical quantization of spacetime.

II. THE AXIAL CURRENT

We start with the generalized definition of the electromagnetic field tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \epsilon_{\mu\nu\alpha\beta}\partial^\alpha B^\beta, \tag{1}$$

where B^μ represents a new gauge field [2]. Maxwell's equations for the fields A^μ and B^μ in Lorenz's gauge ($\partial^\mu A_\mu = \partial^\mu B_\mu = 0$) then reads

$$\partial^\nu F_{\mu\nu} = -\partial^\nu \partial_\nu A_\mu = j_\mu, \tag{2}$$

$$\partial^\nu F^\dagger_{\mu\nu} = -\partial^\nu \partial_\nu B_\mu = g_\mu, \tag{3}$$

where $F^\dagger_{\mu\nu}$ corresponds to $F_{\mu\nu}$'s dual tensor.

The quantity $F_{\mu\nu}$ in (1) is a tensor; $\epsilon_{\mu\nu\alpha\beta}$ is a pseudo-tensor, and therefore the field B_μ must be a pseudo-vector or an axial field. From the point of view of quantum theory the field B_μ represents photon-like particles except for P, T and C parities. In other words, axial photons. From this it follows that (3) is not invariant under time and space reversal, unless g^μ is also a pseudo-vector [3]. This suggests the introduction of an axial current given by

$$g_\mu = g\bar{\psi}\gamma_5\gamma_\mu\psi, \tag{4}$$

where ψ represents a spin 1/2 particle (an axial monopole) with axial charge g.

Since $F^{\dagger}_{\mu\nu}$ is antisymmetric one gets from (3)

$$\partial^{\mu} g_{\mu} = 0, \tag{5}$$

which means axial current conservation and therefore massless axial monopoles.

III. CHARGE QUANTIZATION

Let us consider the gauge-invariant wave function of a charged particle [2] moving in the presence of the axial monopole's field,

$$\Phi_e\left(x, P'\right) = \Phi_e\left(x, P\right) \exp\left[-\frac{\imath e}{2}\int_S F^{\mu\nu} d\sigma_{\mu\nu}\right], \tag{6}$$

S being any surface with contour $P' - P$.

Due to the arbitrariness of the surface S we can write

$$\Phi_e\left(x, P\right) \exp\left[-\frac{\imath e}{2}\int_S F^{\mu\nu} d\sigma_{\mu\nu}\right] = \Phi_e\left(x, P\right) \exp\left[-\frac{\imath e}{2}\int_{S'} F^{\mu\nu} d\sigma_{\mu\nu}\right], \tag{7}$$

which leads to the condition

$$\exp\left[-\frac{\imath e}{2}\oint_{S-S'} F^{\mu\nu} d\sigma_{\mu\nu}\right] = 1, \tag{8}$$

or equivalently to

$$\exp\left[-\imath e\int_V \partial^{\nu} F^{\dagger}_{\mu\nu} dV^{\mu}\right] = 1, \tag{9}$$

where V is the volume involved by the closed surface $S - S'$.

Using (3), we have

$$\exp\left[-\imath e\int_V g_{\mu} dV^{\mu}\right] = 1, \tag{10}$$

which gives

$$Q_V \equiv \int_V g_{\mu} dV^{\mu} = \frac{2\pi n}{e}, \tag{11}$$

n being any integer. Then, using our definition of g_{μ}, eq. (4), we get

$$Q_V = \int_V \left(g\bar{\psi}\gamma_5\gamma_{\mu}\psi\right) dV^{\mu}. \tag{12}$$

As Q_V is a Lorentz scalar, we can perform the calculation in a convenient reference frame. Taking the axial monopole's frame (we can do that formally, even the axial monopole being massless) and using the standard representation for Dirac's spinor,

$$\psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix} = \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \tag{13}$$

we obtain

$$Q_V = g \int_V (\phi^\dagger \, \sigma_i \, \phi) \, dV^i, \tag{14}$$

where σ_i corresponds to the i^{th} Pauli matrix.

Taking now the axial monopole polarization axis in the direction of charge's velocity (positive z-axis, say) we get

$$Q_V = g \int \phi^\dagger \, \phi \, dx \, dy \, dt. \tag{15}$$

Since the axial monopole is massless, the charge's velocity relative to it has to be necessarily 1, the velocity of light. Thus $dt = dz$, and (15) leads to

$$Q_V = g \int \phi^\dagger \, \phi \, dx \, dy \, dz = g. \tag{16}$$

Equations (11) and (16) give

$$\frac{eg}{2\pi} = n. \tag{17}$$

Note that this condition does not depend on the distance between the electric charge and the axial monopole. It implies in charge quantization, in the same way of Dirac's charge quantization condition [1].

IV. THE MASS GENERATION

As we have shown (see (5)), in the present context the axial monopole is necessarily massless, opposed to the massive solitonic descriptions of magnetic monopoles [4, 5, 6, 7]. Let us see what happens if we circumvent such a restriction.

We shall do that through a dynamical mass generation mechanism, by introducing a Higgs scalar field with non-vanishing vacuum expectation value. The Yukawa coupling between the Higgs field and the axial monopole will generate a mass term for the latter, but preserving the axial current conservation.

The free lagrangean for the massless axial monopole is [8]

$$\mathcal{L}_0 = i\bar{\psi}\partial_\mu \gamma^\mu \psi. \tag{18}$$

This lagrangean is invariant under the $U_A(1)$ transformations defined by

$$U_A(1): \; \psi \to e^{i\alpha\gamma_5}\psi, \tag{19}$$

and this invariance leads to the axial current conservation, eq. (5).

Now, we add to this free lagrangean the Higgs and Yukawa terms, to obtain

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{Higgs} - G\bar{\psi}_L \phi_H \psi_R - G\bar{\psi}_R \phi_H^\dagger \psi_L, \tag{20}$$

where ϕ_H stands for the Higgs scalar field, ψ_L and ψ_R are, respectively, the left and right component of ψ, and G is the Yukawa coupling constant.

The lagrangean (20) leads to a massive Dirac equation for the axial monopole wave function ψ, with a mass term given by

$$M = GV, \tag{21}$$

with V standing for the vacuum expectation value of the Higgs field.

It is easy to see that $\mathcal{L}$ is invariant under the $U_A(1)$ transformations (19), provided they transform the Higgs field as

$$\phi_H \rightarrow e^{-i\alpha}\phi_H. \tag{22}$$

Using Noether's theorem we can obtain the conserved current associated to this invariance. It is precisely our axial current (4).

V. DYNAMICAL QUANTIZATION OF SPACETIME

Let us investigate the consequences of the mass generation on the electric charge quantization condition. If the axial monopole is not massless, the charge velocity relative to it, v, is not necessarily 1, and now we have $dt = dz/v$. If we consider an impact parameter sufficiently large, the charge velocity remains unaffected, and from (15) we obtain

$$Q_V = \frac{g}{v}\int \phi^\dagger\, \phi\; dx\, dy\, dz = \frac{g}{v}. \tag{23}$$

Inserting (23) in (11) we have, rather than (17), the condition

$$\frac{eg}{2\pi v} = n, \tag{24}$$

that, again, is independent on the distance between the electric charge and the axial monopole.

The above relation can be satisfied if we simultaneously fulfill

$$\frac{eg}{2\pi} = n_0, \tag{25}$$

and

$$v = \frac{n_0}{n}, \tag{26}$$

with

$$n \ = \ n_0, \ n_0 + 1, \ n_0 + 2..., \tag{27}$$

n_0 being an integer fixed by the values of e and g.

Equation (25) is the charge quantization condition (17), already derived in the massless case. It can be formally obtained from (24) if we consider the limit in which the mass of the particle carrying electric charge vanishes. Or, in another way, if we "switch off" the axial monopole mass, taking the false Higgs vacuum, in which $V = 0$. Physically we do not expect charge quantization to depend on the mass of the particles or on any mass generation mechanism. We shall therefore assume (25) − (27) as the only physical solution of (24).

Condition (26) restricts the values of charge's velocity to rational numbers, a result integrated in theories of discrete spacetime [11]. Furthermore, these rational values form a discrete sequence given by (27). For sufficiently high n_0, this sequence tends to a continuum, except for velocities very near 1, the light's velocity.

If we consider a massive charged particle, equations (26) and (27) lead to an upper limit for the particle's velocity, given by

$$v_0 = \frac{n_0}{n_0 + 1} < 1, \tag{28}$$

since for a massive particle it is impossible to have $v = 1$. For $n_0 \gg 1$, this limit is very near 1.

This limitation of v leads to upper limits for p and E, the momentum and energy of such a particle. For $n_0 \gg 1$ these upper limits are

$$p_0 \approx E_0 \approx m\sqrt{\frac{n_0}{2}}, \tag{29}$$

which are proportional to the particle mass.

The limitation of the energy-momentum space of the particle leads, through the uncertainty principle, to the quantization of its spacetime, with a fundamental length given by

$$a \sim \frac{1}{p_0} \approx \frac{\sqrt{2/n_0}}{m}. \tag{30}$$

The quantization of spacetime here has a dynamical nature, opposed to usual theories of discrete spacetime [12, 13, 14, 15, 16] where it is purely kinematical in origin. Here a fundamental length arises owing to the interaction between the charged particle and the axial monopole. In this aspect, it is akin rather to the pioneer work of Wataghin [17] and to others, more recent, results [18, 19, 20]. Furthermore, the larger the charge mass the smaller the fundamental length a, such that in the classical limit ($m \gg 0$) spacetime will tend to a continuum.

The experimental upper limit $a < 10^{-16}$ cm for the fundamental length of spacetime gives a lower limit for n_0. Inserting that limit in (30), and using for m the electron mass, we get $n_0 > 10^{10}$. Now, from (25) and using for e the electron charge, we obtain a very high lower limit for the axial charge, $g > 10^{12}$.

VI. CONCLUSION

The introduction of the axial current opens up several theoretical perspectives. Well known symmetries in nature, such as space and time reversal, are preserved, and a new symmetry, namely the $U_A(1)$ symmetry, arises in the context of electrodynamics. Charge quantization can be obtained and, in the massive case, it is shown to be intimately connected to a dynamical quantization of spacetime.

But what can we say about the observation of such an entity? Firstly we can say nothing definite about its mass, since the values of G and V in (21) are unknown. However, what is known is that producing massive pairs with large opposite coupling constants is a very difficult experimental task. Moreover, the coupling of the axial monopole to the electromagnetic field is not of vector character, but axial (see (3) and (4)). This means in particular that its production and detection would be directly associated with polarization conditions, which would render its observation nontrivial, even in the massless case.

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